## Linear Algebra

Time: 90 min's

Answer the following questions:

Q1:

- 1- **a**, **b**, **c**, and **d** are vectors,  $\mathbf{d} = [\mathbf{a}^{T} \mathbf{b}^{T} \mathbf{c}^{T}]^{T}$ . If  $\|\mathbf{d}\| = 6.08$ ,  $\|\mathbf{b}\| = 4.12$ , and  $\|\mathbf{c}\| = 3.74$ , **find**  $\|\mathbf{a}\|$ .
- 2- There are 3 vectors  $\mathbf{a} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}^T$ ,  $\mathbf{b} = \begin{bmatrix} 4 & 5 & 6 \end{bmatrix}^T$ , and  $\mathbf{c} = \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}^T$ . Find the **nearest** vector to  $\mathbf{a}$ .
- 3- Find the **angle** between the vectors **a** and **b**, where  $\mathbf{a} = [1\ 2\ 0]^{\mathrm{T}}$  and  $\mathbf{b} = [1\ 0\ -1]^{\mathrm{T}}$ .

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Q2:

1- The table below represents 5 persons' weight and corresponding blood pressure. Find the **correlation type** between weight and blood pressure.

Weight (x)	74	96	85	91	100
Blood pressure (y)	125	140	130	135	147

2- **Find** the value of x, if the vectors  $\mathbf{a} = [1 \ 2 \ 2 \ 3]^T$  and  $\mathbf{b} = [x \ 1 \ -1 \ 1]^T$  are **orthogonal**.

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Q3:

- 1- **Normalize** the vector  $\mathbf{a} = [0 \ 10 \ 20 \ 30]^{\mathrm{T}}$ .
- 2- **Check** if the vectors  $\mathbf{a} = [1 \ 1]^T$  and  $\mathbf{b} = [1 \ -1]^T$  are **basis**. If they are a basis, **expand** the vector  $\mathbf{b} = [5 \ -1]^T$  in this basis.

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Q4:

1- **Solve** the following equations by achieving a **reduced Echelon form**.

$$x_1 + 2x_2 - x_3 = 6$$
$$3x_1 + 8x_2 + 9x_3 = 10$$

$$2x_1 - x_2 + 2x_3 = -2$$

2- **Transform** the following augmented matrices into **Echelon form**, then find the solution if there is one.

a- 
$$\begin{bmatrix} 1 & 2 & -5 & 0 \\ 1 & 3 & -8 & -2 \\ 0 & -3 & 9 & 5 \end{bmatrix}$$

$$b-\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 5 & -2 & 8 \\ 1 & 3 & -1 & 8 \end{bmatrix}$$